TEACHERS' TOLERANCE FOR DISCOMFORT:
IMPLICATIONS FOR CURRICULAR REFORM
IN MATHEMATICS

JEFFREY FRYKHLOM, University of Colorado, Boulder

ABSTRACT. Deficiencies in mathematics achievement in the United States have spurred calls for reform in K–12 curricula and pedagogy. Based on contemporary learning theories, numerous curriculum programs have been developed that emphasize problem solving, critical thinking, mathematical connections, and mathematical communication. These innovations require new—sometimes uncomfortable—roles for both teachers and learners. If students are to construct mathematical knowledge through these curriculum programs, teachers must vitally transform classrooms, model problem solving, explore relevant contexts, and give students time to create, discuss, reflect, hypothesize, and investigate. This kind of teaching necessarily fosters discomfort that is challenging, and often unnerving, for teachers. The research study described here pursued questions related to these facets of curricular reform in mathematics education. In particular, four case studies illustrate various forms of teacher discomfort (as connected to reform-based mathematics curricula) and the impact that uncertainty has on classroom practice in particular, and, more broadly, the likelihood of widespread reform in mathematics education.

Calls for reform in both curricula and pedagogy in mathematics classrooms have been prevalent in the United States throughout the last decade. The urgency of these calls for change was heightened by the United States' performance in the Third Interna-

Teachers' Tolerance for Discomfort

J. Jeffrey Frykholm

Theoritical background

Educational reforms in both curriculum and pedagogy across multiple disciplines have led a growing number of educators to think about the impact of uncertainty and ambiguity on pedagogical practices. With theories of cognitive dissonance as a starting point, these researchers have begun to extend notions of uncertainty and dissonance to the act of teaching. Yet, it is also worth noting that awareness of teacher discomfort is not something new. In an essay on reflective processes in education, John Dewey's acknowledgment of "(1) a state of doubt, hesitation, perplexity, mental difficulty, in which thinking originates and, (2) an act of searching, hunting, inquiring, to find material that will resolve the doubt, settle and dispose of the perplexity" certainly pointed to the kinds of teacher uncertainty emerging in today's mathematics classrooms.

Contemporary scholars are beginning to cast light on these new forms of discomfort. Borrowing from Nancy Angell's work, for example, Villainne notes the "terrible freedom that some teachers experience.


For example, a rich body of research suggests that two dimensions—teachers' beliefs and knowledge structures—strongly affect pedagogy. This research illuminates cases in which teachers' beliefs or knowledge bases impact, align, and perhaps even define their pedagogical practices. Still, the situation may be more complicated than these two dimensions.

As Stephen Brown asserts, the mathematics education community is long overdue in fully addressing the role of uncertainty and discomfort in teaching and learning. Brown advocates "reconstructed visions of teaching" that are capable of "appreciating what it means to be confused, what virtues may follow from it and what kind of pedagogy may make use of it." As Brown notes, Americans have long misconceived discomfort and uncertainty as characteristics that only beginnging teachers should have. To experience such emotions while teaching, is, therefore, a bad sign. Indeed, many American teachers have developed an "ingrained commitment to clarity" that not only is difficult to escape but, in many ways, contradicts the very theories that guide contemporary mathematics teaching and learning. That is, as an appreciation of the social and linguistic nature of mathematics has spread, mathematics teaching...
and learning should "grow through a social and dialectic process in which confusion, uncertainty, and competing conceptions" are inherently present. 6 Contrary to what teachers' immediate, intuitive responses likely may be, the dangerous terrain is not encountered in facing those sources of discomfort but rather in avoiding them.

While we as teachers may create a state of angst by hiding our legitimate mathematically and pedagogical confusions, we run a greater risk of misrepresenting the lived experience of inquiry when we discourage students from honoring their own doubts, ambivalences, disharmonies. No field of inquiry grows in the absence of perceived anomalies; neither does the individual develop on a non-problematic environment. 7

Hence, if teachers are expected to embrace a philosophy of mathematics teaching and learning that no longer rests on a foundation of certainty (that is, to model problem solving and to welcome divergent thoughts and strategies), then educational leaders certainly must endeavor both to understand the kinds of discomfort that will emerge for these teachers and students and to help them work through such discomfort.

RESEARCH CONTEXT, QUESTIONS, AND METHODS

This research occurred in seven schools in five school districts in Colorado, Minnesota, and Wisconsin. Although the districts varied widely in terms of geography, student population, and ethnic and socioeconomic composition, they shared the common feature of being in the early stages of implementing Mathematics In Context, a reform-based curriculum for grades 5 through 8. The general data record included interactions with 25 teachers who had volunteered to participate in the study; 8 were the subjects of detailed case studies.

Several objectives guided this study. Specifically, this research project was designed to (1) identify and explore (and ultimately develop a conceptual framework for) the various domains of discomfort that teachers face; (2) explore dimensions within these various discomfort domains that define whether moments of discomfort for students and teachers become debilitating or educative; (3) explore the impact of discomfort on teachers' beliefs and pedagogical practices; and (4) explore the notion of teaching toward discomfort.

7Ibid.

Mathematics in Context

In the early 1990s, the National Science Foundation began funding curriculum development projects in mathematics education that are now in various stages of implementation. One of these programs, Mathematics In Context, was written by teams at the University of Wisconsin and the Freudenthal Institute in the Netherlands. For several reasons, the implementation of MiC offered a rich context through which to explore the questions of interest guiding this study. For example,

- MiC teachers must engage in the often difficult task of implementing a curriculum that differs significantly from traditional approaches;
- MiC is based on students' explorations of mathematical contexts; the curriculum depends heavily on teachers' ability to let go of the lecturn as they encourage students to conjecture, explore, discuss, investigate, and seek multiple solution strategies;
- MiC's success depends on teachers' ability to tolerate (if not provide) times of uncertainty as students construct knowledge.

These points hint at both the philosophy and the expectations of the curriculum. MiC's basic premise is that students can, and should, "reinvest significant mathematics." To do so, the curriculum encourages teachers not to rush to formalized mathematics (i.e., definitions, procedures, and formulas), but rather to allow students to explore real-world contexts in which significant mathematical topics are embedded. The gradual progression of activities and contexts in the curriculum leads students to the abstract, but in a way that allows them to own the process. Of course, this leads to many questions for teachers: How do I guide this process? How long do I allow students to struggle? When do I intervene with commonly held assumptions and mathematical procedures? How do I assess students in this kind of mathematical experience?

Data Collection, Analysis, and Interpretation

During the first two years of the study, I made at least four multi-day visits to each site. Teachers for whom individual case studies were developed were visited more frequently. Methods of data collection included classroom observations, post-lesson conferences, audiotaped lesson presentations, teachers' reflections as they critically listened to audiotapes of their teaching, ethnographic interviews with teachers, and various artifacts and informal sources of information.
This wide range of data sources was triangulated to investigate the questions that framed this study. Of particular interest were the reflections of teachers as they evaluated audiotapes of their classroom teaching, identified critical moments in their decision-making process, described their feelings and thinking at those moments, and examined the results of their decisions with respect to student engagement and learning. Based on the recommendations of a number of qualitative researchers, the analysis and interpretation of data included the following activities: (1) a continuous reading, reworking, and updating of field and observational notes, (2) a systematic fracturing and coding of the data record, which led to the identification of key themes, categories, and regularities in the data, (3) a memoing process that helped capture informal insights and potential lines of inquiry during data collection and preliminary analysis, and (4) the creation of domain analyses that helped organize emerging codes, relationships, and themes appearing across data sets and case descriptions.

RESEARCH FINDINGS: OUTLINING A FRAMEWORK FOR DISCOMFORT

In an effort to begin to understand (and subsequently identify) various forms of teacher discomfort, a framework has emerged from the data record of this study. As outlined in Table 1, this framework consists of four primary domains of discomfort that appear to be more prevalent in the teaching of reform-based curriculum programs than in the teaching of more traditional programs. Within these domains are dimensions of discomfort. A presentation of more detailed data from the study, as well as excerpts from four of the case studies that were developed, illuminates the four domains. The data both illustrate and support the framework itself as I attempt to extend it as a tool to help understand the landscape of middle school mathematics education reform.

Table 1. Teacher Discomfort Domains and Dimensions

<table>
<thead>
<tr>
<th>Discomfort Domains</th>
<th>Definitions and Dimensions Within Domains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive Discomfort</td>
<td>Cognitive discomfort entails uncertainty over . . .</td>
</tr>
<tr>
<td></td>
<td>Mathematical content knowledge</td>
</tr>
<tr>
<td></td>
<td>Connections between mathematical topics</td>
</tr>
<tr>
<td></td>
<td>Rich mathematical understanding vs procedural (algorithmic) conventions</td>
</tr>
<tr>
<td>Beliefs-Driven Discomfort</td>
<td>Beliefs-driven discomfort emerges when conflicts arise over:</td>
</tr>
<tr>
<td></td>
<td>Beliefs about the nature of mathematics</td>
</tr>
<tr>
<td></td>
<td>Beliefs about how children best learn mathematics</td>
</tr>
<tr>
<td></td>
<td>Beliefs about how mathematics should be taught</td>
</tr>
<tr>
<td></td>
<td>Beliefs about what mathematics should be taught</td>
</tr>
<tr>
<td></td>
<td>Beliefs about the role of mathematics (e.g., as a liter, as a tool, etc.)</td>
</tr>
<tr>
<td>Pedagogical Discomfort</td>
<td>Pedagogical discomfort occurs as a result of:</td>
</tr>
<tr>
<td></td>
<td>New expectations for teaching actions and pedagogical strategies</td>
</tr>
<tr>
<td></td>
<td>New expectations for student engagement in mathematical explorations</td>
</tr>
<tr>
<td></td>
<td>Teachers’ loss of authority (as the former holder of math knowledge) and students’ gain of mathematical autonomy (as they construct their own math knowledge)</td>
</tr>
<tr>
<td></td>
<td>Managing an active learning environment</td>
</tr>
<tr>
<td></td>
<td>Recognizing and acting on connections between teaching, learning, and assessment</td>
</tr>
<tr>
<td>Emotional Discomfort</td>
<td>Teacher emotional discomfort develops as a result of:</td>
</tr>
<tr>
<td></td>
<td>The changing role of the teacher (from “knowledge bearer” to facilitator)</td>
</tr>
<tr>
<td></td>
<td>Loss of role in the classroom (as math experiences change)</td>
</tr>
<tr>
<td></td>
<td>Teachers’ vulnerability because they no longer “know” the mathematics being explored (in contrast to how they used to know the algorithms and procedures they taught)</td>
</tr>
<tr>
<td></td>
<td>Teachers’ vulnerability in becoming active problem solvers along with the students</td>
</tr>
<tr>
<td></td>
<td>Loss of authority to the classroom</td>
</tr>
<tr>
<td></td>
<td>External pressure (parents, administrators, state-level exams, etc.)</td>
</tr>
</tbody>
</table>

Cognitive Discomfort

One of the first comments of many participants in the study related to the degree to which M/C had challenged their mathematical thinking and understanding. The design, structure, and content of the mathematics in M/C depart considerably from more traditional
Jeffrey Perkelobem

Belief-Driven Discomfort

A second significant source of teacher discomfort was connected to the belief frameworks within which all teachers practice their craft. The notion that beliefs affect teaching practices is widely accepted. Explored in a new way in this study, however, was the way that teachers’ beliefs were often entwined in the discomfort they felt while implementing MEC.

MEC is founded on the notion that children not only construct their own understandings of mathematics, but also remain retentive significant mathematics if given a chance. Hence, rather than a static body of truths to be conveyed by the teacher and retained by the learner, the MEC view of mathematics suggests a much broader interpretation of what constitutes mathematics and how it might be best learned. Clearly, this broader notion poses an immediate challenge to teachers who hold more rigid perceptions of the nature of mathematics, as evidenced by a rather frustrated teacher who saw MEC as nothing more than the latest curricular fad.

I have seen things like this come and go. They say it is new and reformed, but in the end, we always come back to teaching basic mathematics again. So why are we going to go through all of this? . . . I know I could teach this way if I wanted to, but the bottom line is that, even though these activities may be fun to do, what our kids need here are the basics—basic math so that they have the tools to be successful in high school. We already have a curriculum here that does that—why change?

Like others, this teacher had a difficult time accepting MEC both because it was difficult to teach and because it was not consistent with her understanding of the nature of middle school mathematics as a collection of basic skills. Similarly, other teachers were forced to reconcile the approach of MEC with their beliefs about how mathematics is learned, and how it should be taught. As one teacher noted,

I always end up teaching this differently than I am probably supposed to because I believe the kids need to see math laid out in a very organized way. . . . I guess I basically don’t think that you can expect kids to grasp everything unless they have a chance to see it very clearly. . . . We always spend the first 15 minutes with a review quiz or some review problems where I can show them what they need to know. I know they need to know those skills, and that is why we practice them.

Two notable points emerge from this quote. First, the teacher believes that basic computational skills were the most essential components of the middle school math curriculum. Second, it appeared
that he believed that teacher exposition is essential for student learning; that is, children learn best not through active construction of knowledge, but rather as recipients of information "laid out" by the teacher. Certainly, this conception is a potentially rich source for discomfort given the constructivist philosophy that guides MIC.

Pedagogical Discomfort

Even if teachers have strong content knowledge and belief structures that are well aligned with the constructivist perspective that guides MIC, they are not guaranteed to be free of discomfort in the MIC classroom. A recurring source of discomfort in this study was related to pedagogical issues that arose in these classrooms—environments that look significantly different from the "traditional" American mathematics classrooms of the 80s and 90s.21

The most prevalent source of pedagogical discomfort arose as teachers grappled with the changing roles required of both teachers and students. First, teachers commented about changes in classroom dynamics. As they diminished their subject-matter authority in the classroom, their students simultaneously became more autonomous as they explored curricular activities and participated in classroom discussions. Often four or five different solution strategies would surface simultaneously. How to manage these moments of uncertainty became a source of discomfort for some teachers. Said one,

At times I marvel at everything they are coming up with. They have so many ideas, and everyone thinks they need to share them. I love to see them so involved, but sometimes I'm just, like, "Help!" I just don't know what to say or do.

As this comment suggests, the teacher was dealing with the difficulty of not only understanding the many solution strategies of her students, but also of simply managing the learning environment. Although certainly not a challenge unique to MIC, managing these disruptive classrooms was unaccustomed for several of the teachers, particularly those with less experience.

Emotional Discomfort

The final discomfort domain is admittedly difficult to disentangle from the previous constructs and certainly needs to be explored with some caution. Yet, the emotional discomfort that appeared to underlie the work and words of many of the teachers in this study may be one of the most important issues facing those concerned about reforming both curricula and pedagogy in mathematics education. Specifically, teachers in this study evidenced their own emotional discomfort, albeit most of the time subtly, over the changing roles they found it necessary to embrace—for both themselves and their students. Teachers hinted at a loss of ritual in the classroom. They felt vulnerable because they no longer "knew" the mathematics at hand as well as they knew the skills and procedures they once taught. They felt uneasy guiding classroom discussions—conversations in which multiple solution strategies, all or none of which could be correct, were regularly presented at the same time. They also spoke eloquently of the vulnerability they sometimes felt as they were forced to become on-the-spot, authentic problem solvers along with their students. When asked what she would do in a situation in which a student presents a strategy that she (the teacher) didn't understand, one teacher responded by suggesting,

Well . . . get stressed! You know—I like to know what I'm doing! . . . (You are going to get kids that are going to come up with something that is the right answer or maybe that isn't even a right answer but you're struggling to explain why it isn't . . .) I like to know what is going on, but I . . . know that I'm not going to have every single answer.

Teachers were similarly burdened by conflicting expectations held in the educational community. In the present era of standards and accountability-based education, teachers and students are under tremendous pressure to perform well on standardized tests that often measure student success on problems that do not look like the contexts of MIC. Although emerging research suggests that students from reform-based programs do not suffer on standardized achievement tests,22 teachers new to innovative curricular programs do not yet have these personal experiences, and they often fear that their students will not have the skill base to do well on the tests or in subsequent high school courses. They report feelings of frustration as they must decide to either trust that the MIC curriculum will in fact prepare their students for successful testing experiences, or take it


upon themselves to supplement MiC with materials they believe will help students succeed on the exams. As one teacher reported,

You give always tell us to "trust" the curriculum. Yeah, that is easier said than done. We don't get to see the big picture—how this will all come together by the end of 8th grade. All I know is that my kids get tested on basic skills this year, and if they don't do well, it falls back on me.

This kind of emotional discomfort, although perhaps not always manifest explicitly in the classroom, nevertheless surfaced repeatedly as teachers talked about the challenges of the new curriculum.

Case Study Episodes

As noted earlier, eight case studies were developed as part of this research study. Highlights of four of these are presented here. These cases were selected because of the contrasts they provide with respect to the discomfort framework under examination. Although some of these teachers appeared to be quite similar in terms of their conceptions and teaching experiences, when faced with moments in the classroom in which the potential for discomfort and uncertainty arose, they responded quite differently. To illustrate these differences and their implications, one specific classroom episode from each of the cases has been described. In all four episodes, the lesson observed came from a geometry unit entitled "Realism"—a unit devoted to area concepts. Although these brief descriptions capture only a snapshot of the more developed case studies, they nevertheless illustrate various dimensions of the discomfort domains.

Case One: Mr. Daniels

Class Context Mr. Daniels taught 6th grade in an upper-middle-class, suburban elementary school. Mr. Daniels has been teaching in the same elementary school for more than 20 years. However, this was the first time in 12 years that he was solely responsible for teaching mathematics. Over the previous years he had team-taught with a math/science specialist.

Mathematical Content Knowledge Mr. Daniels openly stated that he had never enjoyed mathematics as a learner or a teacher, and hence his mathematical knowledge was relatively weak. He said he "struggled" most days just to stay a step ahead of the kids.

Belief Orientation Toward Reform Mr. Daniels openly admitted that he had not thought much about or discussed mathematics education reform of the last decade. Not surprisingly, his beliefs about the teaching and learning of mathematics were fairly fixed and represented a traditional approach to mathematics instruction that was pervasive 15 to 20 years ago. Across interviews, lesson observations, conversations, and a beliefs-survey inventory, Mr. Daniels was more consistent in his beliefs than any other teacher.

Lesson Context Mr. Daniels had just finished the "Realism" unit. On the previous day, he had given his students an end-of-unit assessment. Mr. Daniels reported that although there were a handful of B's on the test, no student had received an A. Thus, Mr. Daniels was using the day to review in preparation for a retest. As was often his pattern, Mr. Daniels began the class with a "mental math" warm-up that consisted of four problems from a traditional text. These problems required the students to recall the formulas for the area and circumference of a circle and the area of a rectangle. Students spent roughly five minutes working these problems independently. Mr. Daniels then spent about 10 minutes discussing the solutions. The class then turned to a review of the answers to the exam questions from the MiC unit.

Description of Lesson Throughout the lesson, Mr. Daniels tightly controlled the conversation in the classroom. Although he did solicit student input, this consisted primarily of short answers for convergent questions. In the second half of the lesson, Mr. Daniels turned his attention to a review of the MiC exam questions. Mr. Daniels presented "the way" to solve each problem at the board, not once acknowledging the possibility of multiple solution strategies. Notably, as the class began to focus on the final problem on the quiz, Mr. Daniels declared that there was a mistake in the answer key in the book for this problem, so he gave you credit even if you got a different answer. He went on to point out that this unit had included several such mistakes in the answer key. As Mr. Daniels began to work the problem in question, however, it became clear that the mistake was not actually in the answer in the book, but rather in Mr. Daniels's solution strategy. As he completed the faulty solution, Mr. Daniels again referred to the wrong answer in the book and moved on without catching his mistake. No students commented on the problem.

Issues Regarding Discomfort On several occasions before this particular observation, and again just before the lesson began, Mr. Daniels stated that his mathematical content knowledge was suspect. Notably, this comment foreshadowed what took place during the ensuing class period. Quite clearly, Mr. Daniels’s knowledge was severely limited by his lack of mathematical content knowledge. His regular reference to the "traditional" textbook, as a supplement for the MiC units suggested that he did not have confidence that the MiC experience was giving students adequate exposure to and practice with the essential skills. Moreover, as illustrated in this particular lesson, he rarely deviated from an expository model of instruction. That is, rarely did
his students have an opportunity to problem solve, discuss, share multiple solutions, and so on. His lack of content knowledge and his traditional conceptions of mathematics teaching and learning were likely causes of his discomfort with the curriculum and the pedagogical strategies it encouraged.

Case Two: Ms. Compton

Class Context. Ms. Compton had five years of teaching experience in 5th and 6th grade classrooms in a middle-class, suburban elementary school. The "Realtime" lesson came fairly early in the fall semester, part of the second MIC unit Ms. Compton had taught.

Mathematical Content Knowledge. Ms. Compton had a strong background in mathematics, including a B.S degree with additional graduate-level credits in mathematics.

Belief Orientation Towards Reform. Despite her rich mathematics background, at the onset of the study Ms. Compton openly doubted reform recommendations such as those contained in the NCTM Standards. She was initially quite reluctant to teach the MIC curriculum because she believed it did not focus enough on procedural skills that she believed her students needed for the 6th grade achievement test. She admitted that MIC would not have been her choice for the new curriculum program, but she had no say in the matter since "it (text adoption) was dropped on us."

Lesson Content. In this particular MIC lesson, students are encouraged to derive the formula for the area of a circle. Students cut 12 congruent wedges from a large circle and then rearrange those pieces in the shape of a parallelogram (see Figure 1). Using their knowledge of the radius and circumference of a circle, the students are to label the length and height of the figure, thereby discovering and deriving the formula for the area of a circle: \( A = \pi r^2 \).

Description of Lesson. Before the class period, Ms. Compton expressed her frustration with the lesson. She remarked,

This lesson is going to be painful. There is no way these students are going to get this on their own. I am going to have to walk them through this by the hand. It is too abstract for them, and besides, they only yesterday were introduced to circumference, diameter, radius, and those things. Math is my thing, but I've never seen this concept before.

Once class began, Ms. Compton shared this bias with her students. She introduced the lesson by stating that what they were going to do today was "very hard. I have no idea how this is going to go. We may not get very far on this lesson."

Despite her pessimism, however, her predictions were not realized. Working in groups, the students not only were able to shape the wedges into a parallelogram, but also recognized the way in which their prior knowledge of the circumference of a circle could be used to help them determine the area of the parallelogram and, in turn, the area of the circle. Almost entirely on their own, each group of students came up with the appropriate solution after much animated discussion. On several occasions as she circulated in the room, Ms. Compton congratulated her students for their good work.

In the post-lesson conference, however, Ms. Compton dismissed much of the success of the lesson by noting her belief that only a small handful of the students actually understood what they did. "I have three or four really bright students that carry everyone else. I'm sure only a few of the kids actually got it."

Issues Regarding Discomfort. Despite her rich mathematics background, and in the face of student successes such as the one just described, Ms. Compton repeatedly discredited the curriculum. The irony about many of her statements (e.g., "the material was too abstract... too difficult for the students... not focused enough on traditional algorithms and procedures") was that they did not appear to reflect the success and understanding attained by her students. Even though her students showed good progress in the development of their mathematical thinking and appeared to thoroughly enjoy the curricular activities, Ms. Compton remained fundamentally at odds with the philosophy and approach of the curriculum. Her primary discomfort with MIC seemed to be connected to her beliefs about mathematics teaching and learning. Because the orientation, content focus, and pedagogical assumptions of the curriculum differed philosophically...
from her position, she had a difficult time not only consistently implementing the curriculum in the spirit that was intended, but also in recognizing the growth her students were experiencing.

Case Three: Ms. Wheaton

Class Content. Ms. Wheaton was a 7th grade teacher with four years of teaching experience at a rural middle school with a large number of students whose primary language was not English.

Mathematical Content Knowledge. Ms. Wheaton did not have a mathematics degree. Despite her lack of background, however, she indicated that her interest in teaching mathematics had grown considerably in the past two years. Ms. Wheaton also suggested that, though she was not afraid of exploring mathematics in depth, she did occasionally bump into the border of her own mathematical content understanding during classroom activity and discussions.

Belief Orientation Toward Reform. Ms. Wheaton repeatedly mentioned her desire to teach in a way that reflected the recommendations of the NCTM Standards, which she had studied in her teacher preparation coursework. She firmly asserted her commitment to learn MiC and to move teaching practices in a direction that would align well with reform-based goals.

Lesson Content. During this lesson, Ms. Wheaton's students were finishing several days of work on tessellations. The primary objective for the class period was for students to create their own tessellations based on prior activities with transformations.

Description of Lesson. Ms. Wheaton's lesson plan was designed in the spirit of the curriculum. She opened class with a short review of tessellations, drawing a number of students into the conversation. After this initial review, Ms. Wheaton instructed the students to create a unique tessellation following the rules for transformations that they had been exploring during the week. The students engaged in the task for about 15 minutes. During that time, Ms. Wheaton interjected numerous comments and shared examples of works in progress as a way to keep the students focused. A sampling of student work shared at the end of the class included examples of various kinds of tessellations that began from different kinds of shapes (triangles, hexagons, rectangles, parallelograms) and included several different kinds of transformations (glide reflections, flips, etc.).

Issues Regarding Discomfort. Ms. Wheaton expressed an interesting contrast to Mr. Daniels. Her content knowledge was, like that of Mr. Daniels, admitted weak. As a result, she was often thrust into moments of uncertainty. Yet, the strength of her convictions to teach MiC as it should be taught allowed her to overcome her discomfort. During this particular class period and in the post-lesson conference, Ms. Wheaton indicated how enjoyable it had been for her to be learning again along with her students. In the post-lesson conference, she said, "Even though I am not sure about some of the math concepts I am supposed to teach, I am learning that if you just go with it, the kids will too. I am sure we have messed up some of it, but most of the time, it feels pretty good at the end." Her openness to confronting her own deficiencies in mathematics in front of her students was evident toward the end of this class period. One of the students had created a rectangular tessellation that had Ms. Wheaton wondering if, in fact, it was a tessellation. She told her student, "I am not sure about this. Your big shapes make a tessellation. But within each of the rectangles, I am not sure if it would still be considered a tessellation. Let's find out." Her last statement about "finding out" was directed at the researcher in the back of the classroom. She shared the picture with the researcher, who confirmed that the picture did satisfy the definitions of a tessellation.

To this confirmation, Ms. Wheaton responded, "Well, I was wrong! So much for that!" Clearly, Ms. Wheaton's openness about her own pursuit of mathematical understanding created a safe environment for students to work through their mathematical uncertainties.

Case Four: Ms. Moore

Class Content. Ms. Moore was a 7th grade teacher in her third year at a rural middle school.

Mathematical Content Knowledge. Ms. Moore had a double major in college—mathematics and history. She spoke openly about her affinity for mathematics and the wide-ranging math background she brought to the classroom.

Belief Orientation Toward Reform. Ms. Moore spoke positively about her preparation experience, noting that two of her professors had actively promoted the reform vision of the NCTM. Ms. Moore reported that, through her preparation experience, she had cultivated strong beliefs about the teaching and learning of mathematics that resonated with the underlying philosophy of MiC.

Lesson Content. During the observed lesson, Ms. Moore’s students were beginning an exploratory activity that asked them to generate numerous strategies for computing and comparing the surface area of two leaves. Students were given various tools (rulers, string, glue, tape, tracing paper, grid paper, etc.) and asked to find as many ways as possible to determine which leaf, if decorated, would require the most chocolate frosting (see Figure 2).

Description of Lesson. Students worked on this initial task in small groups for roughly 20 minutes, while Ms. Moore circulated in the room asking questions of each group. She refused to give them
late. Ms. Moore reported afterward that she had anticipated that this strategy would surface. She did not say anything during or after the presentation, even as several other groups agreed that they had used the same strategy. Two or three students in the room, however, chal-
gen led the presenters. What ensued was a rich discussion about sur-
face area and perimeter in which the students themselves finally all
agreed (by looking at several counterexamples) that measuring the
distance around the leaf with string told them nothing about the
amount of chocolate required to cover it. In the post-lesson confer-
ce, Ms. Moore commented on this discussion.

I knew that if I was silent long enough, somebody in there would have a problem with it. These are smart kids, and they love to argue and disagree
if you give them a chance. And now, because they came up with it on their
own, they own it. I bet not one single student in here would make that mis-
take again after listening to that discussion with their peers.

In subsequent conversations and observations, the notion of teach-
ing toward discomfort appeared repeatedly in the data record for
Ms. Moore.

IMPLICATIONS: TOWARD A THEORY OF DISCOMFORT

The episodes presented here show various manifestations of
teacher discomfort that resulted in, ultimately, vastly different expe-
riences for students in the classroom. Yet, making links between
teacher discomfort and learning opportunities for students is not easy.
These four cases illustrate interesting anomalies. On one hand, Ms.
Compton professed to bring strong mathematical understanding to
the classroom, but nevertheless had minimalist views of both the
curriculum and her students, resulting in fairly convergent teaching.
On the other hand, Ms. Moore used her confidence in her content
knowledge to create open-ended learning opportunities in which
students generated and directed much of the discourse. In a parallel
comparison, both Mr. Daniels and Ms. Wheaton admitted that their
mathematical content knowledge was limited and freely talked
about resulting moments of uncertainty in the classroom. Yet how
they engaged students and the learning they saw, “the only way they are going to un-
derstand math is if they wrestle with it.” I try to push them like this
as much as possible. They have to figure it out for themselves.” An
excellent example of this point occurred as one of the groups sug-
gested a solution that (wrongly) assumed a relationship between
perimeter and surface area. The students demonstrated how to trace
the outline of each leaf with a piece of string. They then suggested
that the leaf with the longest string would require the most cho-
co...
to think carefully about the impact that beliefs and knowledge in particular have upon teacher uncertainty.

Deliberating Discomfort. Based on these findings, one might expect deliberating discomfort to emerge to the context of several possible scenarios. A clearly recognized case would be when a teacher simply does not have sufficient content knowledge to engage students in the mathematics of the curriculum. The severity of this discomfort, which is likely increased when such teachers also have beliefs that run counter to the philosophy upon which the curriculum is based. A second easily identifiable case would be that of a teacher who, irrespective of content knowledge, believes strongly that the scope, design, and philosophy of the curriculum are inappropriate for student learning. Given the power of belief structures upon practice, this case virtually assumes some level of discomfort in the classroom and a likely departure from the intended pedagogy of the curriculum. A third category of cases relates to emotional and pedagogical influences upon teacher discomfort. Although pedagogical and emotional discomfort has many possible causes, it is likely that teachers who are struggling with such forms of discomfort are doing so at least in part because of some uncertainty in either their knowledge of the math at hand or their beliefs about the nature of teaching and learning mathematics.

Educative Discomfort. As implied previously, not all discomfort in the classroom is inherently negative. The notion of "instructive" discomfort—or perhaps "educative" discomfort—is equally important to develop within this framework. What is most notable about teachers who have an ability to tolerate discomfort—so not use it as a pedagogical tool—is the alignment of their belief structures with the philosophy of the curriculum. As suggested by the cases of Ms. Wheaton and Ms. Moore, regardless of a teacher's knowledge base, discomfort can become educative as long as teachers believe the uncertainty is a necessary and natural component of learning. Unlike teacher knowledge structures, teachers' beliefs seem necessary for a healthy implementation of MIC. And, when coupled with strong content knowledge as well, it would appear from this research that the ingredients for successful implementation are in place, and that teachers might begin even to lead their students toward discomfort in order to promote deeper learning.

Links to Theory: Discomfort and Self-Efficacy

The idea of discomfort is certainly not new to the educational community. This particular elaboration of teachers' tolerance for discomfort may, in fact, be better understood if we draw upon important contributions from other bodies of research. One such connection is the work that has been done on teachers' sense of self-efficacy. Bandura suggests that individuals possess a self system that enables them to exercise a measure of control over their thoughts, feelings, and actions...Beliefs that people have about themselves are key elements in the exercise of control and personal agency. That is, individuals' and in this case, teachers' reflections and thought processes mediate experiences and actions. Therefore, as Pajares notes, "Knowledge, skill, and prior attainments are often poor predictors of subsequent attainments because the beliefs that individuals hold about their abilities to control these outcomes almost always influence the ways in which they will behave." Bandura has referred to such beliefs about personal agency as self-efficacy—beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situations. Bandura also suggests that individuals rely on their sense of self-efficacy to regulate activity by engaging in tasks that they believe they can competently navigate and avoiding those in which they lack confidence or understanding. It would seem, therefore, that teachers' self-efficacy may have more to do with adventurous teaching than other factors that are often cited as impediments to innovative teaching (e.g., content knowledge, belief systems, prior experiences, etc.). For, as Pajares notes, efficacy beliefs help determine how much effort people will spend on an activity, how long they will persevere when confronting obstacles, and how resilient they will prove in the face of adverse situations— the higher the sense of efficacy, the greater the effort, persistence, and resilience.

When taken together with the data gathered in this study, these theories of self-efficacy call for at least an initial attempt to elaborate the relationship between self-efficacy and teachers' tolerance for discomfort. Figure 3 represents one interpretation of this potential in-


\[Rid.

\[Albert Bandura, Self-Efficacy: The Exercise of Control (New York: W. H. Free-


\[David Cohen, "Teaching Practice: Plus ça change . . .", in Contribut-

\[ing to Educational Change Perspectives on Research and Practice, ed. P. Jackson


\[Frank Pajares, "Self-Efficacy Beliefs in Academic Settings," Review of Educa-


[Jeffrey Prykholm]
SUMMARY

This article articulates a framework for considering the impact of teacher discomfort on pedagogical practice. This research points toward a theoretical foundation that may provide the grounding for more thorough examination of the complex and overlapping layers of teaching that affect successful implementation of reform-based mathematics curricula. The mathematics education community has produced a host of effective and rich curricular programs that hold great promise. Before us now, however, is the challenge of thoroughly examining the new set of complex pedagogical issues that accompany these new curricula.

This study, therefore, is central to our understandings of growth in teachers and teaching practice. It examines the essence of constructivist-based teaching, those critical moments in the classroom when children and teachers may meet—when the teacher’s willingness to let go overlaps with the child’s native curiosity, when teachers pose open-ended questions that foster creative ideas and strategies, when the teacher’s willingness to be vulnerable meets the child’s desire to trust, when the teacher’s decision to remain silent allows children to develop their own voices, when the teacher’s appreciation for the elegance of mathematics leads the child to lifelong learning and appreciation of mathematics.

These are magical moments that transform both teachers and learners. Yet, we know precious little about the intuitions that lead teachers to these places, and how those intuitions can be fostered. This article has been an attempt to lay the groundwork for greater exploration of these powerful moments so that we can begin to more effectively help teachers facilitate learners’ mathematical power through excellence, and perhaps even adventurous, teaching.

JEFFREY PYSKOLM is Assistant Professor of Education at the University of Colorado at Boulder, Box 249, Boulder, CO 80309-2249; phone: (303) 492-7749; e-mail: sylvakholm@colorado.edu.

Figure 3. Self-Efficacy as a Mediator of Teachers’ Tolerance for Discomfort

Figure 3 illustrates the concept of self-efficacy as a mediator in the relationship between teachers’ perceptions of the value of an innovative curriculum and their tolerance for discomfort. This diagram outlines the potential categories of discomfort, including emotional, pedagogical, cognitive, and belief factors, and shows how self-efficacy acts as a filter to influence teachers’ tolerance for discomfort.

The diagram includes the following elements:
- Innovative Curriculum Context and Pedagogy
- External support for program
- Teachers’ perception of program value
- Potential Categories of Discomfort
- Emotional
- Pedagogical
- Cognitive
- Belief
- Self-Efficacy Filter
- Teachers’ Tolerance for Discomfort
  - Debilitating discomfort
  - Educative discomfort
  - Fidelity of Implementation

The diagram highlights the interaction between teachers’ perceptions of the value of innovative curricula and their tolerance for discomfort, moderated by self-efficacy. This framework provides a basis for understanding how teachers respond to discomfort and how self-efficacy plays a crucial role in mediating this response.